# Relaxation from color Compton Scattering of Plasmons in A Collisionless Quark-Gluon Plasma *

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**Abstract**

Non-Abelian kinetic effects are taken into account for the study of the relaxation of collective motion in a quark-gluon plasma(QGP). An explicit Compton scattering is considered to calculate the relaxation time. It is shown that the new non-Abelian relaxation has a physical mechanism from the non-linear dynamics and the meaning of the relaxation is discussed.

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The quark-gluon plasma(QGP) has been predicted to be produced in high energy nuclear collisions. The properties of QGP are of importance for understanding experimental results[1]. Collective motion plays an importance role in a plasma. Remarkably, the collective behaviors of a QGP have difference from a electromagnetic plasma due to color degrees of freedom[2]. It means that some new transport problems emerge, which is associated with color degrees of freedom[3, 4]. Obviously, the relaxation processes from collisional term used to be easily realized since we are familiar with the old knowledge. The collisional terms of QGP thus were given by Selikhov and Gyulassay[5] under the consideration of the quantum fluctuations and the color relaxation from the color collisional term which is only relevant to color degrees of freedom was studied as well as momentum relaxation[6]. However, we can't help but ask such questions: Is there a relaxation related to both color and momentum

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space? If there is, what is the relaxation? In this letter, we will try to solve this problem. Firstly, we need to review the development of QGP kinetic-theory approach. It is well-known that kinetic theory[7] is believed to describe correctly the quark-gluon plasma(QGP) physics as well as temperature field[8] and kinetic-theory approach to transport coefficients is of advantage of convenience. In fact, more and more attention is recently concentrated on the applications of kinetic theory to hot QCD[6, 9]. However, the treatment of non-Abelian counterparts in kinetic equations had ever been a difficult task[10, 11] for long time. We just believed that the solution of the problem had been made improvement until the non-Abelian mean-field dynamics[11] and 'double perturbation' approach[12] were presented recently. These progresses are also of importance for understanding color relaxation physics. We roughly give the analysis before new physics are studied in detail: Apart from collisional terms, non-Abelian covariant derivative enters the formalism of the kinetic equations. This includes color self-coupling contribution in the covariant derivative. Here it is clear that the self-coupling term represents the collective behaviors due to color degrees of freedom instead of dynamic behaviors described in collisional term in color space. Then we think that a relaxation process from the non-linear dynamics shall also be produced even if for a collisionless QGP.

Now we start our studies from the kinetic equations of a collisionless QGP[13],

$$p^\mu D_\mu Q_\pm(\mathbf{p},x) \pm \frac{g}{2} p^\mu \partial_p^\nu \{F_{\mu\nu}(x), Q_\pm(\mathbf{p},x)\} = 0, \tag{1}$$

$$p^\mu \tilde{D}_\mu G(\mathbf{p},x) + \frac{g}{2} p^\mu \partial_p^\nu \{\tilde{F}_{\mu\nu}(x), G(\mathbf{p},x)\} = 0, \tag{2}$$

where the letters with $\tilde{}$ represent the corresponding operators in adjoint representation of SU(3). Here we consider the density fluctuations $\Delta Q_\pm, \Delta G$ deviating from equilibrium distribution functions $Q_\pm^{(0)}$ and $G^{(0)}$ and replace the induced field $A$ by $a$. Assuming the fluctuations to be weak, $p \sim gT, a_\mu \sim T, i\partial_\mu \sim gT$. Thus we write the equations for the fluctuations from 'double perturbation' approach[12]

$$p^\mu \partial_\mu \Delta Q_\pm + ig \sum_\lambda p^\mu [a_\mu, \Delta Q_\pm^{(\lambda)}] \pm g p^\mu f_{\mu\nu} \partial_p^\nu Q_\pm^{(0)} = 0 \tag{3}$$

$$p^\mu \partial_\mu \Delta G + ig \sum_\lambda p^\mu [\tilde{a}_\mu, \Delta G^{(\lambda)}] + g p^\mu \tilde{f}_{\mu\nu} \partial_p^\nu G^{(0)} = 0 \tag{4}$$

where $\lambda$ denotes the powers of induced field. In Ref[12], we had showed the above perturbation equations do not break non-Abelian gauge symmetry. Marking the summation terms by $S_\pm$ and $\tilde{S}$, we give

$$
\begin{aligned}
&+\quad ig^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\delta(k-k_1-k_2-k_3)\frac{1}{p\cdot(k_2+k_3)p\cdot k_2}[p\cdot a(k_1),[p\cdot a(k_2),p^\mu f_{\mu\nu}(k_2)]]\\
&+\quad \cdots\cdots) \qquad (5)
\end{aligned}
$$

$$
\begin{aligned}
\tilde{S} = &-\int\frac{d^4k}{(2\pi)^4}\partial_p^\nu G^{(0)}\left(ig\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\delta(k-k_1-k_2)\frac{1}{p\cdot k_2}[p\cdot\tilde{a}(k_1),p^\mu\tilde{f}_{\mu\nu}(k_2)]\right.\\
&+\quad ig^2\int\frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4k_3}{(2\pi)^4}\delta(k-k_1-k_2-k_3)\frac{1}{p\cdot(k_2+k_3)p\cdot k_2}[p\cdot\tilde{a}(k_1),[p\cdot\tilde{a}(k_2),p^\mu\tilde{f}_{\mu\nu}(k_2)]]\\
&+\quad \cdots\cdots) \qquad (6)
\end{aligned}
$$

We move the summation terms to the right-hand side of the equations, a quasilinear formalism of the kinetic equations is obtained

$$
p^\mu\partial_\mu\Delta Q_\pm \pm gp^\mu f_{\mu\nu}\partial_p^\nu Q_\pm^{(0)} = -ig\sum_\lambda p^\mu[a_\mu,\Delta Q_\pm^{(\lambda)}] \qquad (7)
$$

$$
p^\mu\partial_\mu\Delta G + gp^\mu\tilde{f}_{\mu\nu}\partial_p^\nu G^{(0)} = -ig\sum_\lambda p^\mu[\tilde{a}_\mu,\Delta G^{(\lambda)}] \qquad (8)
$$

Especially note that an illustration of the above equations is here essential: In previous works, the nonlinear terms in the right-hand sides of the equations were removed because of the linearization treatment of non-Abelian covariant derivatives in the kinetic equations[4, 6, 14], so that only the relaxation from collisional terms were discussed. Now we effectively regard the right-hand side terms as 'collisional terms', but do not consider truly collisional terms. It is the terms lost in past works that will give rise to a considerably large physical effect, which will be seen later. In relaxation-time approach, we have identities from (5),(6),(7)and (8)

$$
p^\mu u_\mu\nu_\pm\Delta Q_\pm = -igS_\pm \qquad (9)
$$

$$
p^\mu u_\mu\nu_g\Delta G = -ig\tilde{S} \qquad (10)
$$

where $\nu$ is the equilibration rate parameter and $u_\mu$ is the hydrodynamic velocity which describes the motion of the plasma as a whole. After the Fourier transformation, the equations (9)and(10) are respectively multiplied by $\mathbf{v}\cdot\mathbf{a}(\omega,\mathbf{k}')$ and $\mathbf{v}\cdot\tilde{\mathbf{a}}(\omega,\mathbf{k}')$ and then the mean values are taken with respect to statistical ensemble and momentum space. Thus one finds in the plasma rest frame:

$$
\nu_\pm p_0\int\frac{\mathrm{d}^3p}{(2\pi)^3}\langle\Delta Q_\pm(k)\mathbf{v}\cdot\mathbf{a}(k')\rangle = -g\int\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{Im}\langle S_\pm(k)\mathbf{v}\cdot\mathbf{a}(k')\rangle, \qquad (11)
$$

$$
\int\frac{\mathrm{d}^3p}{(2\pi)^3}\langle\Delta G(k)\mathbf{v}\cdot\tilde{\mathbf{a}}(k')\rangle \quad \int\frac{\mathrm{d}^3p}{(2\pi)^3}\mathrm{Im}\langle\tilde{S}(k)\mathbf{v}\cdot\tilde{\mathbf{a}}(k')\rangle \qquad (12)
$$

where $\mathbf{v} = \frac{\mathbf{p}}{p_0}$ is the velocity of plasma particle. One should note that for a baryonless plasma the numbers of quarks and antiquarks are equal to each other and $\nu_+ = \nu_-$ and a effective equilibration parameter$\nu_{\rm eff}$ is defined via[14]

$$\nu_{\rm eff} = \nu_+ \frac{N_f}{N_f + 2N} + \nu_g \frac{2N}{N_f + 2N}. \tag{13}$$

In general, it is sufficient that the first two terms in $S$ are remained, the first term of which represents 3-wave correlator and be believed to vanish[15]. So only the terms for $\lambda = 2$ are required to be calculated here. Then we obtain the relaxation time formula as

$$\begin{aligned} \frac{1}{t_{pc}} &= \nu_{\rm eff} \\ &= g^2 N \frac{\int \mathrm{d}\mathbf{v} \frac{\mathrm{d}k_1}{(2\pi)^4} \pi \delta[\omega_1 - \omega - (\mathbf{k}_1 - \mathbf{k}) \cdot \mathbf{v}] (\frac{\omega_1}{\omega_1 - \mathbf{k}_1 \cdot \mathbf{v}} - \frac{\omega}{\omega - \mathbf{k} \cdot \mathbf{v}}) (\frac{(\mathbf{k}_1 \cdot \mathbf{v})^2}{\mathbf{k}_1^2} \langle a_l^2(k_1) \rangle + \frac{(\mathbf{k}_1 \times \mathbf{v})^2}{\mathbf{k}_1^2} \langle a_t^2(k_1) \rangle) \langle \mathbf{v} \cdot \mathbf{a}(k) \rangle}{\int \mathrm{d}\mathbf{v} \frac{\omega}{\omega - \mathbf{k} \cdot \mathbf{v}} \langle \mathbf{v} \cdot \mathbf{a}(k) \rangle} \end{aligned} \tag{14}$$

where $l$ and $t$ respectively denote the longitudinal and transverse components of the field.

Clearly, the delta-function $\delta[\omega_1 - \omega - (\mathbf{k}_1 - \mathbf{k}) \cdot \mathbf{v}]$ indicates the a Compton scattering process of two collective modes(plasmons)in the phase space with both color and momentum degrees of freedom. To clarify the scattering process, the explanations follow: First, both color and momentum exchanges arise in the Compton scattering process; Second, the color exchange differs from that without momentum; Finally, the Compton scattering is also not usual Compton scattering in momentum space. Therefore, a non-Abelian Compton scattering (or called color Compton scattering) mechanism shall be defined. We also know that the scattering process is from the long-range interactions and then $\omega > |\mathbf{k}|$ for any thermal collective mode from the dispersion relation given by the past works[7, 13]. So we obtain in long-wavelength limit approximation

$$\frac{1}{t_{\rm pc}} = 0.236227 g^2 T. \tag{15}$$

The order in $g$ is between the momentum relaxation $\frac{1}{t_{\rm p}}$ being of order of $g^4 {\rm ln} g^{-2}$ and color relaxation $\frac{1}{t_{\rm c}}$ being of order of $g^2 {\rm ln} g^{-1}$[4, 6].To make $t_{\rm pc}$ clear, a analysis is necessary:

1. Color relaxation denoted by $t_{\rm c}$ is from static limit[6], i.e., $\mathbf{v} = 0$. It shall be a purely dynamic effect from collision term only considering color degrees of freedom[3]. While what we are studying is the non-Abelian Compton scattering of two collective modes which is influenced by both color degrees of freedom and plasma particle motions($\mathbf{v} \neq 0$). So the relaxation process denoted by $t_{\rm pc}$ shall be a kinematic effect with color dynamics. We can simply call it momentum-color-relaxation abbreviated to pc-relaxation here.

2.Since the pc-relaxation is related to both color and motion, the relaxation process will

was predicted to vanishes in semiclassical approximation[4] and $\frac{1}{t_{\rm p}}(\sim g^4T\log g^{-2})$ for the perturbative QCD plasma[3] is of lower order than $\frac{1}{t_{\rm pc}}(\sim g^2T)$. Then pc-relaxation is of important interest in the semiclassical limit domain.

3. The result in static limit can also be obtained from the non-Abelian mean-field dynamics based on kinetic-theory[11]. There it was believed that the non-perturbative dynamics of soft gluons at leading logarithmic order can be described[11, 16, 17]. It maybe implies that pc-relaxation instead of color-relaxation controls the color diffusion coefficient in high temperature QGP.

4. The pc-relaxation process shall be produced due to the non-linear dynamics from (3)-(6). We also find from (14) that two-mode scattering or 'two-quasiparticle collision' is the physical mechanism of the pc-relaxation. The effective collision, in fact, is the long range interaction.

In conclusion, we find a kinetic effect on color and momentum relaxations from analysis of non-Abelian kinetic theory. After that, we study the relaxation process and evaluate the relaxation time. We think what we get in this letter is of benefit to deeply understanding and fully receiving non-Abelian transport problem.

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